Calculation of Permeability Tensors for Unstructured Grid Blocks

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Geostatistical models of reservoir properties can be hundreds of millions of cells; it is impossible or inefficient to use them directly in flow simulation due to computational cost. Upscaling techniques are applied to average the fine scale permeability up into flow simulation grid blocks. In cases where unstructured grids are used or the geology inside the grid is not aligned with the grid geometry, full permeability tensors arise instead of a diagonal tensor. The focus of this work is on development of a method to characterize the full permeability tensor for an unstructured grid block using fine scale heterogeneity information. A single phase flow-based upscaling is performed and a prototype program called `ptensor` *is developed based on the random boundary conditions and optimization technique. The permeability tensor (full, symmetric and diagonal) is calculated for 2-D and 3-D grids and some sensitivity analysis is performed.*

Introduction

Geostatistical modeling of petrophysical properties generally provides fine scale models with hundred millions of grids. Feeding these fine scale models to flow simulation is impractical due to the computational inefficiency. The most powerful available simulators can handle up to million grids. On the other hand, large scale geostatistical modeling lead to smooth model of reservoir properties and this may imposed the inaccuracy to the flow simulation results. Upscaling techniques are often considered to average the fine scale models to coarser scale models while preserving the fine scale heterogeneity. A simple averaging is sufficiently and reasonable for variables that average linearly; however, in the case of permeability which does not average linearly, a simple arithmetic averaging is inadequate.

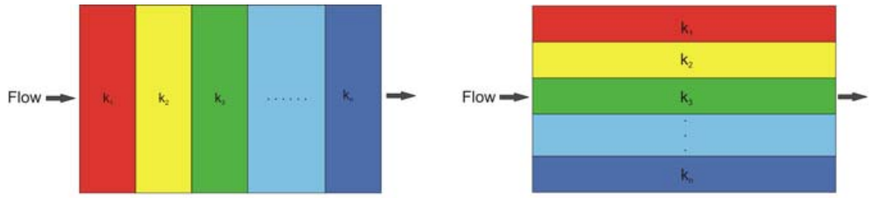
Commonly unstructured grids are used in order to better capture the flow response near complex reservoir features such as faults and wells. Irregularly shaped grids do not conform to the underlying fine scale model and this irregularity changes the assumption of simulators which consider that the pressure equation has a diagonal permeability tensor. Flow Simulation on unstructured grids requires directional permeability or full tensor permeability to be specified. Recent modifications to flow simulators permit the solution of the flow equations using a symmetric tensor. However, calculating the permeability tensor for unstructured grid is challenging.

The motivation of this research is to find a simple, fast and accurate method to generate permeability tensor for unstructured grids. We present a new method to calculate full, symmetric or diagonal tensor for any corner point geometry grids. The unstructured grid is surrounded by a bounding box and the geometry is simplified with the fine grids. The steady state flow equation is solved (Finite Difference) for the fine grids within the bounding box and the results are used to calculate the permeability tensor of corresponding coarse regular or irregular grid. Randomly assigned boundary conditions are used and the results are optimized to get the desired full, symmetric or diagonal tensor. A GSLIB-style code is developed to implement upscaling to general 2-D or 3-D grids.

Background

Permeability upscaling refers to a procedure in which the underlying fine scale permeability is averaged up to return the effective permeability of a larger domain. There are several upscaling techniques available in literatures. Some of them are useful in some specific conditions while others are more general and appropriate for complex grid geometries and medium.

In an ideal case, the simple averaging methods are fast and easy to implement. For example, the equivalent permeability for a group of fine grid blocks serially arranged has been analytically proven to be equal to their harmonic average. If the blocks are arranged in parallel to the flow direction, the equivalent permeability is equal to their arithmetic average (see Figure 1) (Deutsch, 1987; Kelkar and Perez, 2002). These two basic cases are the worse and the best case of connectivity in the direction of flow, respectively.



$$k_{eff} = \frac{n}{\sum_{i=1}^{n} \frac{1}{k_i}} \qquad k_{eff} = \frac{1}{n}\sum_{i=1}^{n} k_i$$

Figure 1. Upscaled effective permeability for simple cases of series (left) and parallel (right) layers. redrawn from Kelkar and Perez (2002).

Gomez-Hernandez and Wen (1994) showed that a geometric mean is useful but not in the case of completely heterogeneous medium.

According to the work of Cardwell and parsons (1945), Journel (1986), Deutsch (1989) and Desbarats and Dimitrakopoulos (1990) showed the application of power averaging in estimation of equivalent block permeability. They realized that the power, ω, depends on the number of factors such as heterogeneity at small scale, the block shape and flow conditions inside the block.

$$k_V = \left[\frac{1}{V}\int_V k_\omega^p(\mathbf{u})\right]^{\frac{1}{p}}$$

where k_v is the coarse block permeability, V is the coarse block volume, k_ω is fine scale permeability and $p \in [-1,1]$ depends on the number of factors such as heterogeneity at small scale, the block shape and flow conditions inside the block.

King (1989) used the renormalization technique to compute the block permeability. In this method the upscaling is started from a small block and the size of block is increased successively until the flow simulation block size is reached. This method is very fast and simple but it does not work very well in highly anisotropic media.

For more complex cases with increasing heterogeneity, flow-based upscaling techniques yield more accurate results. In this type of upscaling the flow equation is solved for pressure and the results are used to obtain the block permeability. Choosing appropriate boundary conditions is very important in flow-based upscaling. Figure 2 Shows examples of boundary conditions which are commonly used in literatures.

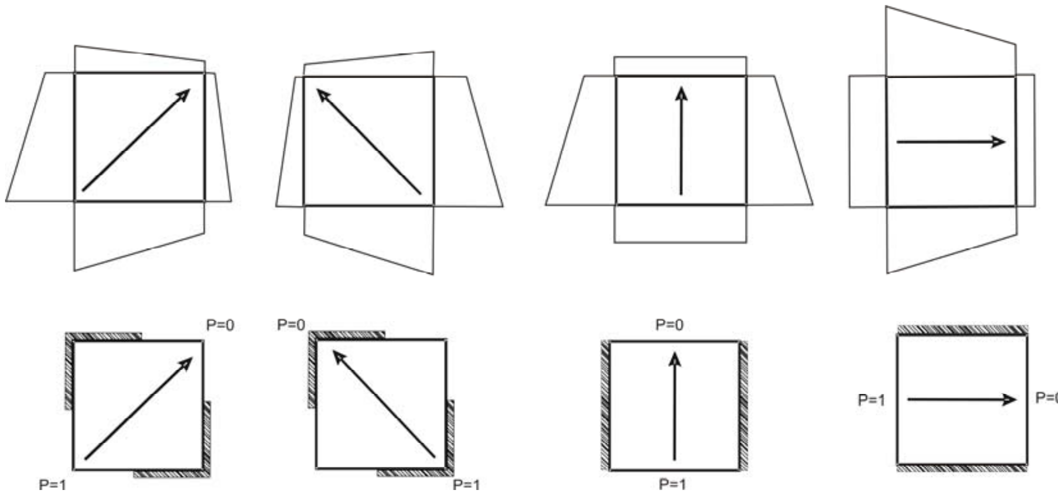


Figure 2. Linear boundary conditions (Gomez-Hernandez, 1994) in top row, constant pressure and no-flow boundary conditions (White and Horne, 1987) in bottom row. Hatch lines show the no-flow faces.

Warren and Price (1961) applied this technique with constant pressure and no-flow boundary conditions for regular coarse grids to obtain the diagonal tensor. Usually cases that involve the use of irregular grid or heterogeneous permeability field at fine scale require calculating the full permeability tensor. White and Horne (1987) were the first to propose a technique to determine full non-diagonal block permeability tensors. They used different sets of boundary conditions and solve the flow equation for entire field of study. The resulting block permeability tensors are not always symmetric or positive-definite. Gomez-Hernandez (1994) used many linear boundary conditions and solved the flow equations over an area comprising the coarse block and a skin region. They got full tensor for regular coarse grids. Durlofsky's (1991) idea of periodic boundary returns symmetric and positive definite full permeability tensor in medium with periodic condition (repetitive geological structures).

Almost all of the above mentioned techniques are applied on regular grids. Tran (1995) proposed a method in which the pressure is calculated in the smallest rectangle that includes the irregular block. However, the calculated permeability is diagonal. He (2000 and 2004) applied Durlofsky's periodic boundary condition and solved the flow equations with a finite element method for general quadrilateral grids. This method gives the same accuracy as finite difference method in 2-D quadrilateral grids but it is not efficient for 3-D grids. Prevost (2003) implemented both permeability and transmissibility upscaling on general 3-D grids and showed that the transmissibility upscaling is generally more accurate but it is time consuming because the flow equation must be solved for all grids interfaces.

Methodology

Flow based upscaling technique is used to calculate effective permeability of coarse block. Consider single rectangular block (2-D) or a regular box (3-D) imposed on a fine scale model. The idea here is to calculate the pressure at fine scale with specific boundary conditions applied at the boundary of the coarse block and then use the solution to calculate the full permeability tensor for that coarse block. In order to calculate pressure at fine scale, single phase steady state flow equation with the assumption of incompressible fluid and rock is considered:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial P}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial P}{\partial z}\right)=0$$

where P is the pressure, and k_x , k_y and k_z are the fine scale permeability in x, y and z direction respectively.

To get more robust and realistic tensor, general linear boundary condition is considered. In this technique four pressures in 2-D or eight pressures in 3-D are randomly assigned to the corners of the coarse grid. In 2-

D cases the pressure gradient at each edge are considered to change linearly between to pressures at two corners. In 3-D, each face will have different pressure distribution. This pressure distribution is defined by using pressures at four corners and applying the bilinear interpolation. This will provide a smooth map of pressure at six sides of a 3-D coarse block (Figure 3).

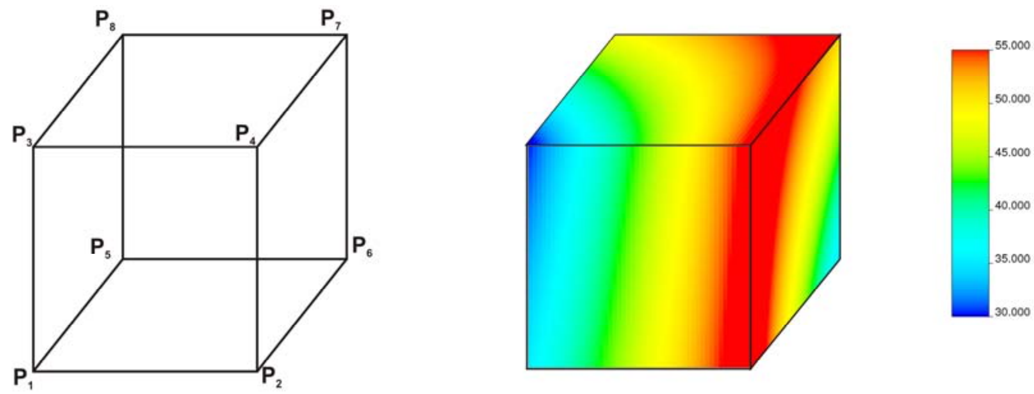


Figure 3. Eight pressures are randomly assigned to the corner (left) and example of pressure distribution generated by bilinear interpolation (right).

Given sets of random boundary conditions, the flow equation is solved for the all fine girds inside the coarse grid and the pressure differences and flowrates are calculated. The pressure differences are calculated by getting the average of all fine scale pressures over the coarse block volume. The flowrates are the volume average of flowrates between the fine grids:

$$\Delta P_x = \frac{1}{n_{tot}} \sum_{k=1}^{n_z} \sum_{j=1}^{n_y} \sum_{i=2}^{n_x} \left(p_{i,j,k} - p_{i-1,j,k} \right)$$

$$\Delta P_y = \frac{1}{n_{tot}} \sum_{k=1}^{n_z} \sum_{i=1}^{n_x} \sum_{j=2}^{n_y} \left(p_{i,j,k} - p_{i,j-1,k} \right)$$

$$\Delta P_z = \frac{1}{n_{tot}} \sum_{j=1}^{n_y} \sum_{i=1}^{n_x} \sum_{k=2}^{n_z} \left(p_{i,j,k} - p_{i,j,k-1} \right)$$

where $p_{i,j,k}$ is the fine scale pressure in grid with i, j and k index.

The flow rates are the volume average of flow rates between the fine scale points:

$$Q_{actual-x} = \frac{1}{n_{tot}} \sum_{k=1}^{n_z} \sum_{j=1}^{n_y} \sum_{i=2}^{n_x} T_x^{i-1/2,j,k} \cdot \left(p_{i,j,k} - p_{i-1,j,k} \right)$$

$$Q_{actual-y} = \frac{1}{n_{tot}} \sum_{k=1}^{n_z} \sum_{i=1}^{n_x} \sum_{j=2}^{n_y} T_y^{i,j-1/2,k} \cdot \left(p_{i,j,k} - p_{i,j-1,k} \right)$$

$$Q_{actual-z} = \frac{1}{n_{tot}} \sum_{j=1}^{n_y} \sum_{i=1}^{n_x} \sum_{k=2}^{n_z} T_z^{i,j,k-1/2} \cdot \left(p_{i,j,k} - p_{i,j,k-1} \right)$$

where $n_{tot} = n_x.n_y.n_z$ and $T_x^{i-1/2,j,k} = 2.\frac{k_x^{i-1,j,k}.k_x^{i,j,k}}{\left(k_x^{i-1,j,k} + k_x^{i,j,k}\right)}.\frac{\left(\Delta y.\Delta z\right)}{\Delta x}$ is the transmissibility term.

Optimization

Appling many different boundary conditions and having sets of pressure differences and flowrates; we can fit a tensor with the minimum error using an optimization technique. In this technique we start with an initial assumed tensor. The predicted flow rates are calculated using the following general Darcy's law.

$$\begin{bmatrix} Q_x^* \\ Q_y^* \\ Q_z^* \end{bmatrix} = -\begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \cdot \begin{bmatrix} \Delta p_x \\ \Delta p_y \\ \Delta p_z \end{bmatrix}$$

An initial objective function (error) is calculated by using the following formula:

$$Obj = \frac{\sum_{B.C.} \sum_{dir} \left\| Q_{act} - Q^* \right\|}{\left(No.\ of\ \ Boundary\ Conditions \right) \times \left(No.\ of\ directions \right)}$$

where the inside summation is the flow rate taken over the directions (X,Y and Z) and the outside summation is over different boundary conditions.

We proceed by proposing a change in tensor and calculating the new objective function. The change is kept if the new objective function is smaller than the previous one. This process is repeated many times until reaching the desired error. There is a flexibility to fit full, symmetric or diagonal tensor by adding a constraint for the cross terms in tensor to be equal (symmetric) or zero (diagonal).

Dealing with Unstructured Grid

The methodology discussed above can also be applied for unstructured grids. The coarse irregular grid is surrounded by the smallest rectangle (2-D) or smallest box (3-D). We can discretize the irregularly shaped coarse scale block using the underlying fine scale model. This can be easily determined by evaluating whether or not the centre point of the fine scale block falls inside the irregular coarse block. The pressure differences and flowrates are then averaged over the fine grids which are inside coarser grid. A more accurate approximation to the irregular coarse scale block can be obtained if the underlying heterogeneity model is at a sufficiently fine scale.

In 2-D cases, any polygonal shape can be easily handled. In this work for 3-D cases the corner point geometry is considered. The unstructured grid is defined by two sets of corner points which indicated the upper and the lower planes. Both upper and lower planes should have the same number of vertices in a way that each corner point in the upper plane has the corresponding points in the lower plane. Two upper and lower planes are considered to be connected by planar surfaces. The grids are not necessarily aligned vertically and can be tilted in some direction to conform to the geology structures. Figure 4 shows examples of corner point grid.

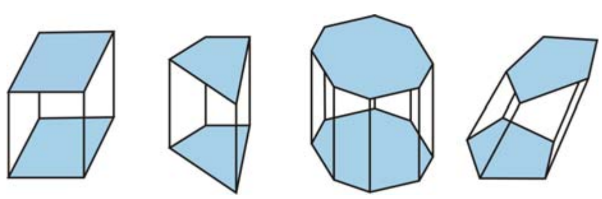


Figure 4. Example of corner point structured (left) and unstructured grids (middle and left). The areal cross section is a convex polygon.

Sensitivity Analysis

The permeability tensor calculated by flow based upscaling techniques is not unique and depends on many factors. Flow boundary condition, size of bounding box, the fine scale permeability assigned to the buffer zone and the geological features inside the coarse grids are examples of factors which affect the permeability tensor. Each factor is discussed separately in below.

Flow Boundary Condition

It has been shown that the block effective permeability is not only depends on the fine scale heterogeneity but also on the flow boundary conditions. Different pressures at boundary impose different flow regimes inside each grids and this affect the value of permeability which is numerically calculated or experimentally measured. This becomes more important when considering the permeability tensor which has cross-flow terms. Generally, linear pressure gradient and no-flow boundaries are considered. However, this is not what realistically happening in reservoir. In this work we considered random pressures assigned at the corner of coarse grid so the flow may not be imposed in a specific direction and this will help us to appropriately capture the effect of cross-flow in coarse block. Since many different boundary conditions are needed for this method, the minimum number of boundary conditions required to get stable permeability tensor is an issue. This can be approximately determined by the following example.

In this example we considered a synthetic permeability model with a single 3D grid imposed on it. Ten different permeability realizations are generated by SGSIM (Figure 6). These models are considered as the fine scale permeability in *X* and *Y* direction (k_x and k_y). The permeability in *Z* direction is considered multiple of k_x by a constant factor. The permeability tensor is calculated for different number of random boundary conditions varying from 3 to 50. Variation of permeability tensor and the minimum error are compared with the number of boundary conditions. The main diagonal tensor components and the error are become stable for approximately 10 boundary conditions (Figure 5).

In order to find the minimum number of boundary conditions for each case, the stabilizing criterion is defined as follows:

$$\sum_{i=1}^{3}\sum_{j=1}^{3}\left|\frac{k_{i,j}^{l}-k_{ref_{i,j}}}{k_{ref_{i,j}}}\right|<1\% \qquad l=3,....,50$$

where k is the permeability tensor, i and j indicate the row and column of the permeability tensor and k_{ref} is the tensor calculated for 50 boundary conditions (more stable). The minimum l which satisfies this condition is considered as the minimum number of boundary conditions.

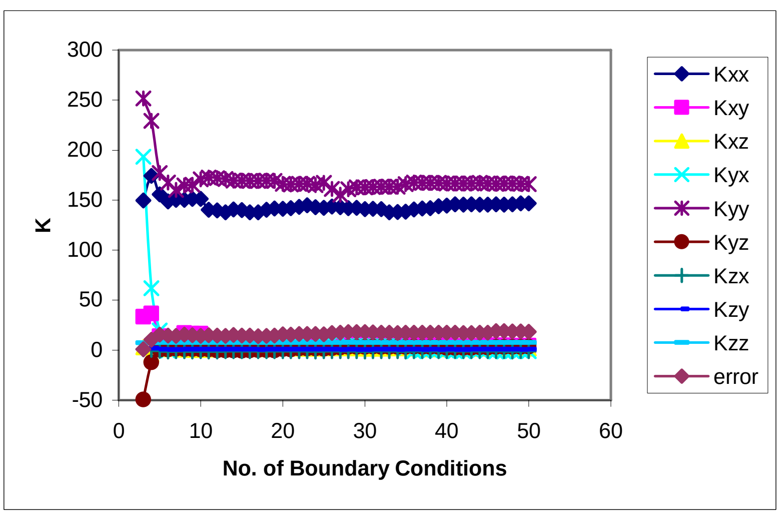


Figure 5. The permeability tensor variation with number of random boundary conditions for realization 8.

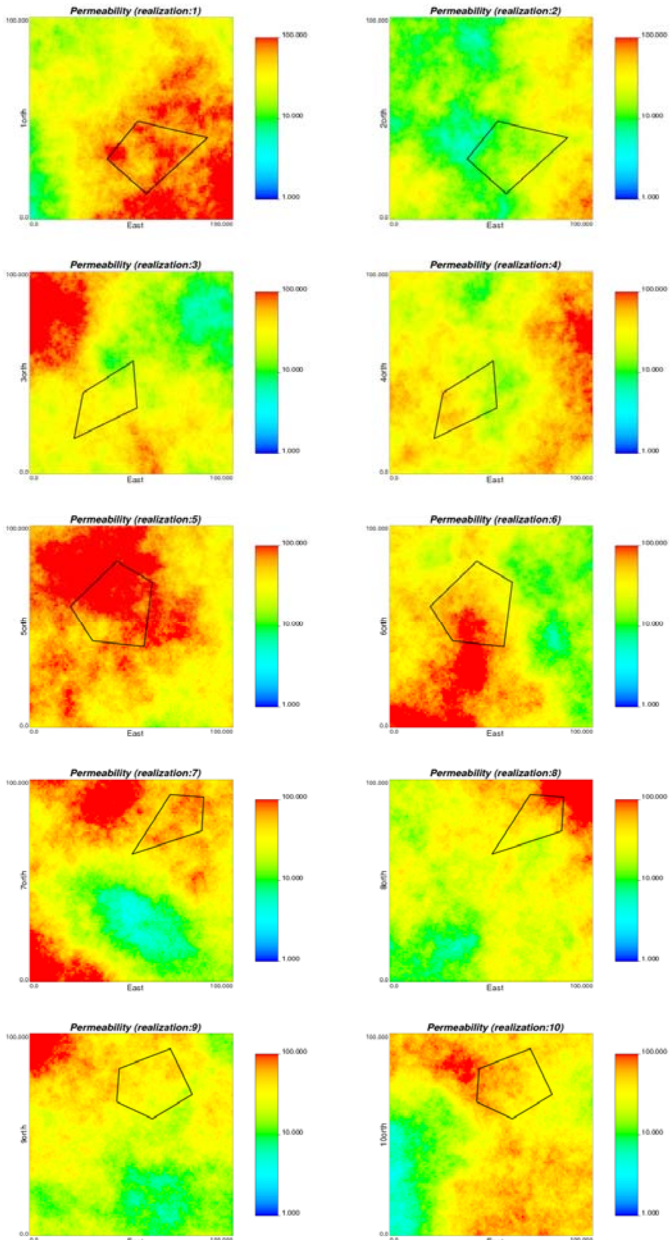


Figure 6. Permeability model and the 3-D grid considered in this example (top) and the permeability tensor variation with number of random boundary conditions (bottom).

Bounding Box

In flow-based upscaling the flow equation is usually solved locally for specific grid. Here we considered a bounding box around the unstructured grid which the flow equation is solved for that box. Two factors may affect the value of permeability tensor for a grid surrounded by bounding box; Size of bounding box and the permeability value assigned to the fine grids which are inside of bounding box and outside of irregular grid (buffer zone). We discuss more with the following example.

A 3-D synthetic permeability model is generated (100 x 100 x 50 grids). A single unstructured grid is considered on this field. The bounding box for this grid comprises of 20 x 20 x10 fine grids. The full permeability tensor is calculated for the coarse grid considering three cases with different bounding box size. Smallest bounding box (20 x 20 x 10), the bounding box expanded at each side by half of the smallest box size (40 x 40 x 20) and the bounding box expanded at each side by same size of smallest box (60 x 60 x 30).See Figure 7.

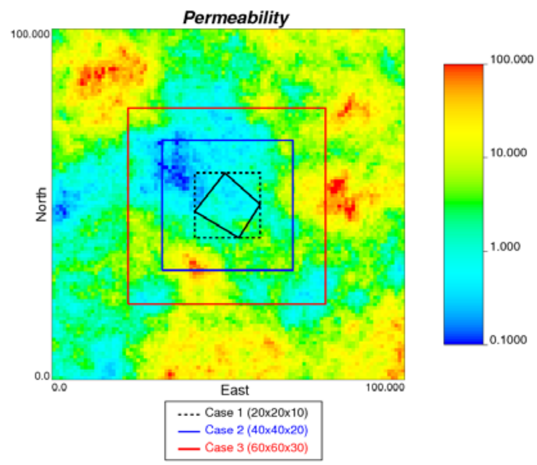
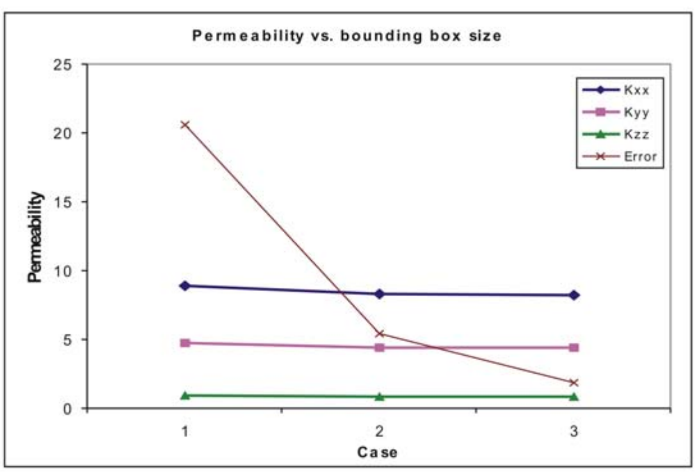


Figure 7. Three different bounding box sizes considered for this example.

According to the plot of Figure 8, as the bounding box size increases the permeability tensor component in main diagonal (k_{xx}, k_{yy}, k_{zz}) decreases slightly but the error drops significantly. However, if comparing the computing time for these three cases, larger bounding box is not efficient to be considered.



Case	CPU Time (s)
1	3.1
2	12
3	99.27

Figure 8. Variation of permeability tensor with bounding box size.

In the above example we considered heterogeneous buffer zone. In order to check the sensitivity of permeability to buffer zone permeability, the example is run for a large bounding box (case 3) and permeability values of 1, 10, 20, 100 and 500 mD. As Figure 9 shows, the permeability tensor calculated for an unstructured grid of this example increases as the buffer zone permeability increases. The error is also increases systematically as buffer zone permeability increases.

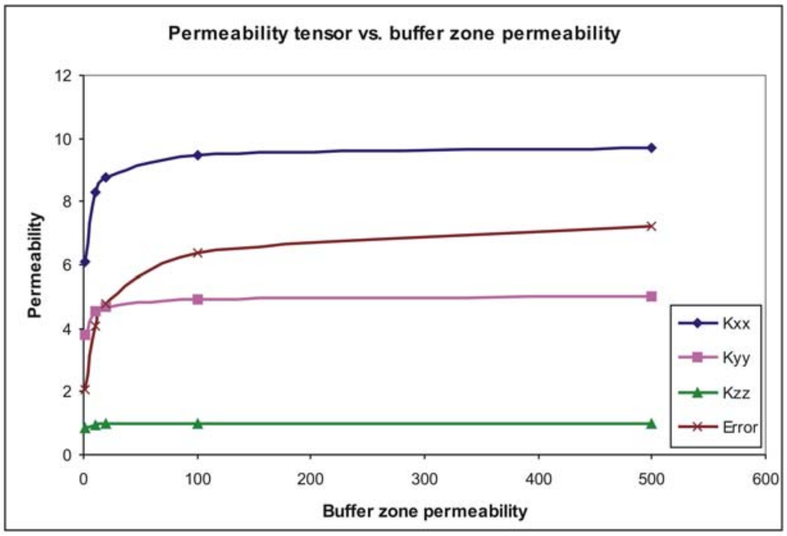


Figure 9. Variation of permeability tensor with buffer zone permeability.

Geological Features

The fine scale heterogeneity makes significant effect in the coarse grid permeability. In some cases the geological features are not aligned with the coarse block sides. This may lead to cross flow term appears. We discussed this more with the following example.

Consider a single grid in the middle of a field. Sequential Gaussian simulation is run for a square field and the permeability is modeled for different continuity directions (azimuth 0° to azimuth 90° with increment of 5°). Figure 10 shows the permeability models and the single grid located at the center of field. Full permeability tensor is calculated for each case.

As we physically expected the permeability in *X* direction increases and the permeability in *Y* direction decreases when the orientation of geological features changes from 0° to 90°. In order to better realize the variation the cross-flow terms we define cross-flow index as follows:

$$\textit{cross-flow index} = \frac{\left(k_{xy} + k_{yx}\right)}{\left(k_{xx} + k_{yy}\right)}$$

As we can see in Figure 11, the index is smaller when the flow is dominated in the *X* or *Y* direction and this happens when there is more connectivity in those directions.

Grid Orientation

Sensitivity of permeability tensor to the grid orientation is checked by an example. A permeability model is selected from the previous example (azimuth:90°). A single pentagonal grid block is imposed in the middle of the field. The grid is rotated and the permeability tensor is calculated for each case. Figure 12 shows 12 cases with the same permeability model and different grid orientations (0° to 360° azimuth in increments of 30°). The results show that the permeability tensor components changes when the grid orientation changes. However, the components do not vary with a specific relation.

Full, Symmetric and Diagonal Permeability Tensor

The same permeability model and unstructured grid as the example considered for the effect of bounding box is considered with a large bounding box (60x60x30) and 20 random boundary conditions. The full, symmetric and diagonal permeability tensors are fitted. The results are shown in Figure 13.

The results showed that the full tensor introduces a fitting error of less than %2 (for this example) and the error increases when symmetric or diagonal tensor is fitted. This example reveals the fact that using full permeability tensor returns the flowrates which are more similar to the actual flowrates.

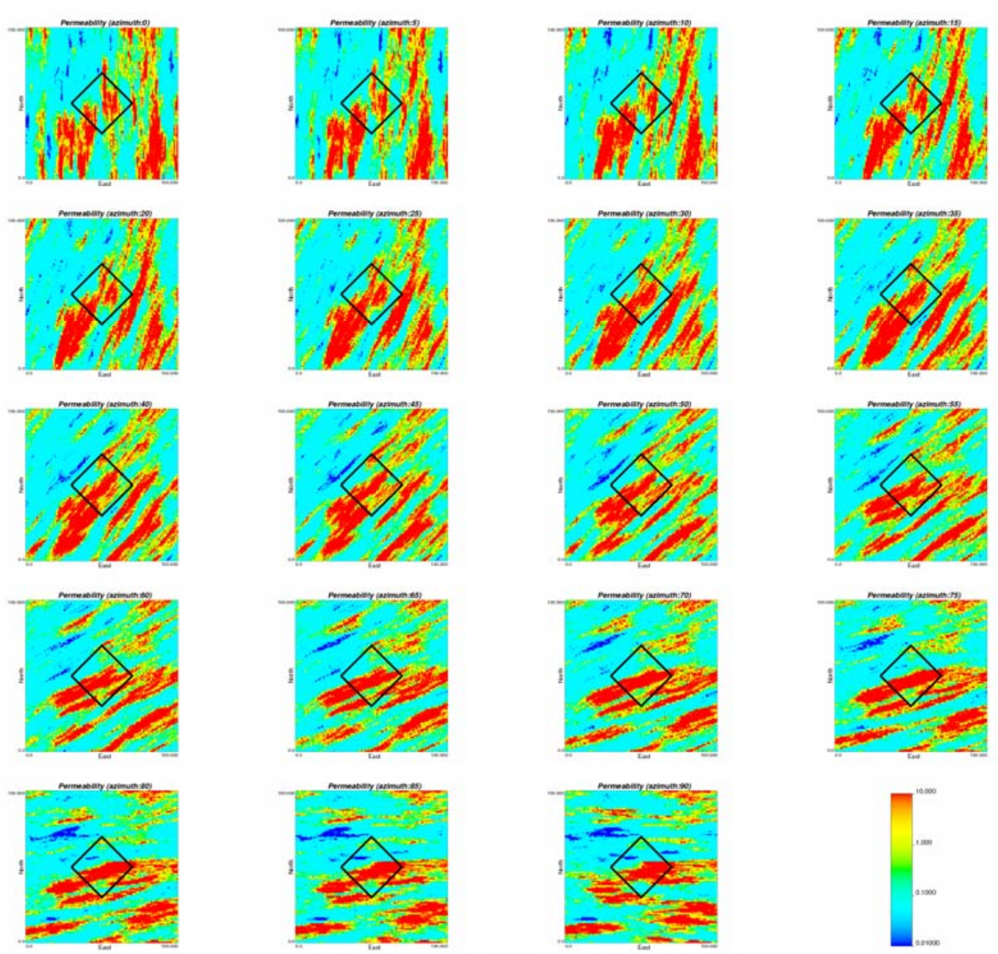


Figure 10. Permeability models with different direction of continuity and the location of grid.

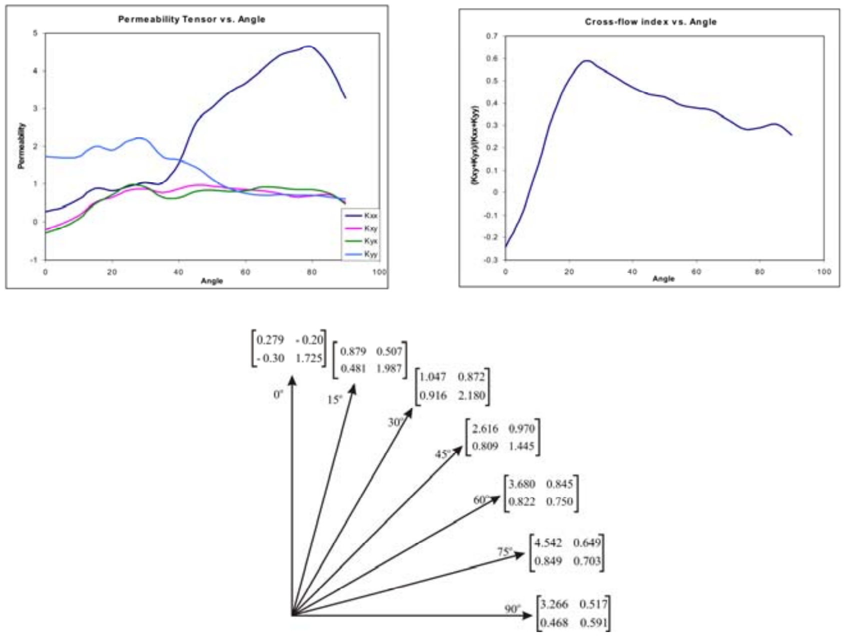
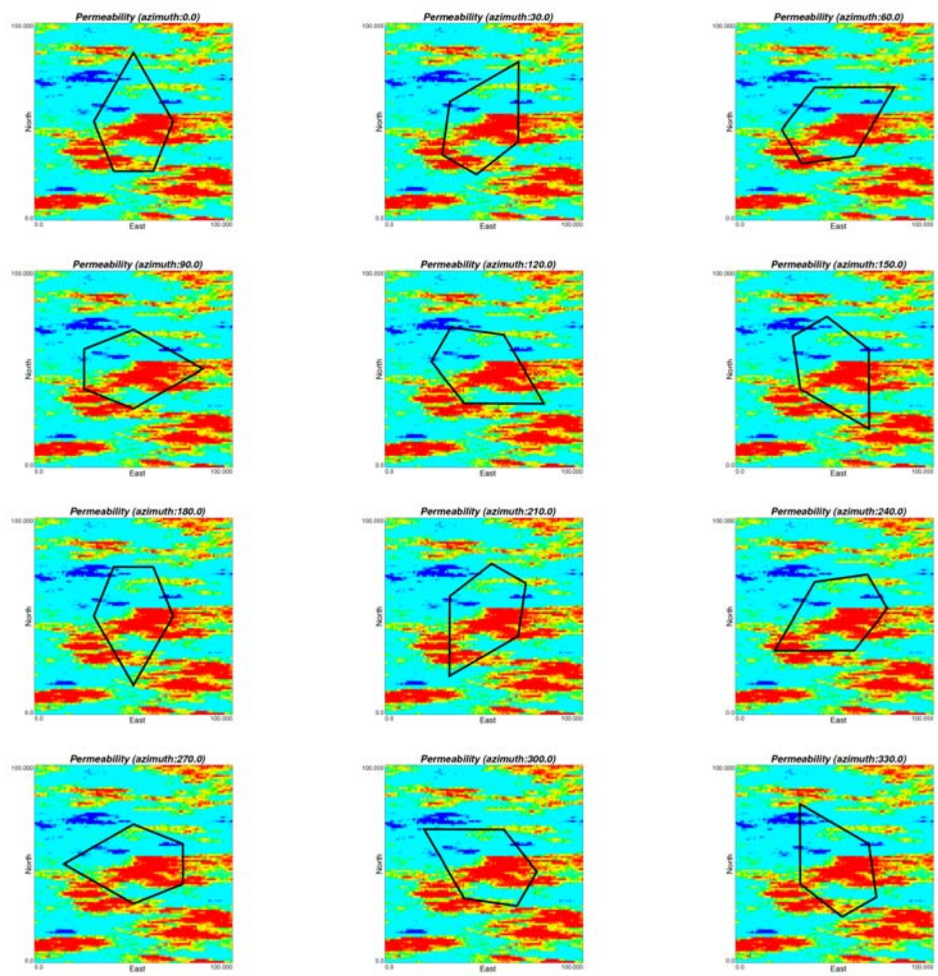


Figure 11. Variation of permeability tensor with direction of continuity in fine-scale permeability model.



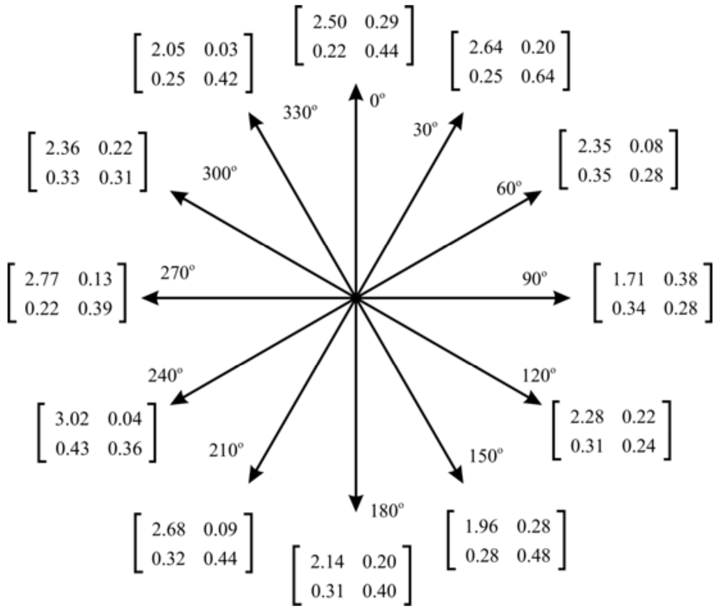


Figure 12. Variations of permeability tensor with grid block orientation.

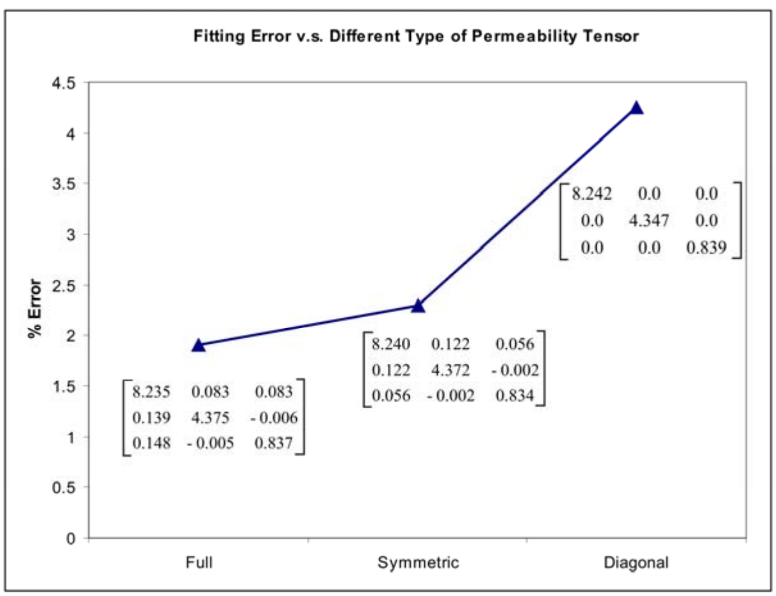


Figure 13. Fitting error for full, symmetric and diagonal permeability tensor.

Conclusions and Future Work

A flow based technique is developed to calculate permeability tensors for unstructured grid blocks. Multiple flow simulations with random boundary conditions are considered and optimization is used to calculate the effective permeability tensor. The resulting tensor is positive definite and it can be full, symmetric or diagonal. Results are quite sensitive to the boundary conditions, bounding box conditions and the fine scale heterogeneity model. The `ptensor` code is reasonably fast however the computation time increases as the grids become larger. A sensitivity analysis was performed; however, the methodology should be validated for a complete unstructured grid model. This can be done by running flow simulation on both fine scale and the upscaled permeability model, and comparing simulation results such as recovery factor.

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Appendix: Fortran Code (`ptensor`)

The GSLIB-style program, called `ptensor`, is developed to calculate the permeability tensor for 2-D and 3-D unstructured grid block. It is based on the finite difference solution of single-phase, steady-state flow equation and optimization method. The `ptensor` program permits different options related to the shape of coarse grid (Polygon in 2-D or corner point geometry in 3-D), size of the buffer zone, the permeability value inside this zone, and an option to fit full, symmetric or diagonal tensor. The parameters required for this program are:

```
                          Parameters for PTENSOR
                         ************************
Line START OF PARAMETERS:
 1   2                      -2-D (X/Y) or 3-D (X/Y/Z)
 2   perm.dat               -input data faile with permeability
 3   1 2 3 0 0 0            -columns for kx,ky,kz,ky/kx,kz/kx
 4   50  50  10             - input : nx, ny, nz
 5   1.0 1.0 1.0            - input : dx, dy, dz
 6   ptensor.out            -file for permeability tensor output
 7   ptensor.dbg            -file for debugging output
 8   usg.dat                -file for unstructured grid
 9   1                      - Number of grids
10   1 1 1 1 1 1            -Buffer zone size:X-left,X-right,Y-left,Y-right,Z-left,Z-right
11   1                      -Buffer zone : Homogeneous(1), Heterogeneous(0)
12   20                     - if (1),constant permibility value
13   20                     -Number of Boundary Conditions
14   69069                  -random number seed
15   1                      -fitting option: 1=diag, 2=symm, 3=full
16   0.001  5000            -minimum objective function, max per
```

In the first line there is an option to choose the dimension. Both 2-D and 3-D cases are acceptable. The information about the fine scale permeability data file are input in Lines **2** and **3**. The number and size of grid cells in the input file should be specified in Lines **4** and **5**. In Line **6** and **7,** the name of output and debugging files are specified. The debugging file comprised of the flow simulation results for each coarse grid. The file for unstructured grids is specified in line **8**. In unstructured grid file the number of corner points for each grid is provided first. In the following lines, the coordinates are specified by two triples of X, Y and Z coordinates, representing two corresponding points in upper and lower planes (description of 3-D grids are described in the paper). Below is an example of acceptable unstructured grid file.

```
Unstructured grid file
4
 53.2    48.6    7       53.2    48.6    3
 38.2    30      7       38.2    30      3
 57.4    12.9    7       57.4    12.9    3
 87.5    40.5    7       87.5    40.5    3
5
 8       10      7       8       10      3
 20      10      7       20      10      3
 29      6       7       29      6       3
 25      5       7       25      5       3
 10      5       7       10      5       3
```

Six integer numbers in line **10** control the size of buffer. Each integer value shows the number of fine grid which should be added to the smallest rectangular region. For example “2 2 2 2 2 2” means that the bounding box should be expanded by two fine grids at each side. There is an option for the permeability value of buffer zone. Lines **11** and **12** enable the user to choose if the buffer zone is homogeneous or heterogeneous and what is the homogenous permeability value is (if put 1 in line 11). In the line **13**, the number of random boundary conditions for which the flow simulation should be solved is specified. The random number seed is specified in line **14**. It should be a large odd number. Three options are available to fit the tensor (full, symmetric or diagonal). Line **15** corresponds to these options. The minimum objective function (error) and the number of iterations for tensor fitting are specified at the last line.